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Also solved by A. F. Carpenter, G. B. M. Zerr, A. H. Holmes, J. Scheffer, and G. W. Greenwood.

Professors Zerr, Greenwood, and Scheffer, and Mr. Holmes denoted the angles at the center subtended by the sides, by 2θ, 2ϕ, and 2ψ, and showed that these angles are equal to 120° each. Professor Carpenter showed that the inscribed triangle with one side constant and of maximum perimeter is isosceles, and then showed that of all isosceles triangles inscribed in a circle, the equilateral has the maximum perimeter.

240. Proposed by L. MORDELL, Philadelphia, Pa.

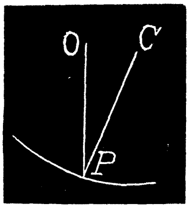
Show that the osculating conic of the catenary $y=c\cosh\frac{x}{c}$ at the point for which $y=\frac{c\sqrt{10}}{2}$ is a parabola.

I. Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Let O be the center of curvature of the point considered, and C the center of the conic of closed contact, then (*vide* Joseph Edwards' *Differential Calculus*, where the problem is proposed as an exercise):

$$\frac{\cos\phi}{R}=\frac{1}{\rho}-\frac{\partial\phi}{\partial s},\ \text{and}\ \ \phi=\tan^{-1}\tfrac{1}{3}.\frac{\partial\rho}{\partial s},$$

where $\phi=\angle OPC$, $OP=\rho=$radius of curvature, s an arc of the given curve, and $PC=R$. From



$$y=\frac{c}{2}(e^{x/c}+e^{-(x/c)}),\ \text{we find}\ \frac{\partial y}{\partial x}=\tfrac{1}{2}(e^{x/c}-e^{-(x/c)}),$$

$$\frac{\partial^2 y}{\partial x^2}=\frac{1}{2c}(e^{x/c}+e^{-(x/c)},\quad \frac{\partial s}{\partial x}=\tfrac{1}{2}(e^{x/c}+e^{-(x/c)}).$$

$$\therefore\rho=\left[\left(\frac{\partial s}{\partial x}\right)^3\Big/\left(\frac{\partial^2 y}{\partial x^2}\right)\right]=\frac{c}{4}(e^{x/c}+e^{-(x/c)})^2.\qquad \therefore\frac{\partial\rho}{\partial s}=\frac{\partial\rho}{\partial x}.\frac{\partial x}{\partial s}=e^{x/c}-e^{-(x/c)}$$

therefore, $\phi=\tan^{-1}\frac{e^{x/c}-e^{-(x/c)}}{3}$, or, $e^{x/c}-e^{-(x/c)}=3\tan\phi$.

$$\therefore\frac{\partial\phi}{\partial x}=\frac{(e^{x/c}+e^{-(x/c)})\cos^2\phi}{3c}.\quad \therefore\frac{\partial\phi}{\partial s}=\frac{\partial\phi}{\partial x}.\frac{\partial x}{\partial s}=\frac{2}{3c}\cos^2\phi.$$

For the value $y=\frac{c}{2}\sqrt{10}$, we find $\tan\phi=\frac{1}{3}\sqrt{6}$, $\cos^2\phi=\frac{3}{5}$, $\rho=\frac{5c}{2}$, $\frac{\partial\phi}{\partial s}=\frac{2}{5c}$; substituting in $\frac{\cos\phi}{R}=\frac{1}{\rho}-\frac{\partial\phi}{\partial s}$, we find $\frac{\sqrt{\frac{3}{5}}}{R}=\frac{2}{\rho c}-\frac{2}{\rho c}=0$; $\therefore R=\infty$, and since the conic, the center of which is at an infinite distance, is a parabola, the assertion is proved.

II. **Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.**

In the catenary, $y=c\sec\psi$, we have $s=c\tan\psi$ and $\rho=c\sec^2\psi$.

If a conic whose center is C osculates a curve at a point P, the center of curvature for the curve at that point being O, we have

$$\tan\phi=\tfrac{1}{3}.\frac{d\rho}{ds}, \text{ and } \frac{\cos\phi}{PC}=\frac{1}{\rho}-\frac{d\phi}{ds}, \text{ where } \phi=\angle OPC.$$

[Edwards' *Differential Calculus*, p. 356, 3d edition.]

$$\rho=c(1+\tan^2\psi)=c+\frac{s^2}{c},\quad \frac{d\rho}{ds}=2\tan\psi.$$

$$\therefore \tan\phi=\tfrac{2}{3}\tan\psi;\ (1+\tan^2\phi)d\phi=\tfrac{2}{3}(1+\tan^2\psi)d\psi.$$

$$\therefore \frac{\cos\phi}{PC}=\frac{1}{\rho}-\frac{d\phi}{d\psi}.\frac{1}{\rho}=\frac{1}{\rho}\left(1-\frac{6(1+\tan^2\psi)}{9+4\tan^2\psi}\right)=\frac{3-2\tan^2\psi}{\rho(9+4\tan^2\psi}=0,$$

when $y=c\sec\psi=\dfrac{c\sqrt{10}}{2}$.

Hence (since $\cos\phi\neq0$), PC is infinite, and the conic is a parabola.

This question was set at Oxford in 1889.

Also solved by G. B. M. Zerr.

MECHANICS.

200. **Proposed by C. N. SCHMALL, 89 Columbia Street, New York City.**

An elastic string whose weight is W is laid over the top of an inclined plane so as to remain at rest. Determine how much the string will be elongated, knowing, M=modulus of elasticity, L=normal length of string, and ϕ=inclination of the plane.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let M=mass of unit length of the unstretched string. The equation of equilibrium is $dT+Mg\sin\varphi ds=0$. But $MgL=W$.

$$\therefore T+\frac{W\sin\varphi s}{L}=\text{constant.}\quad \text{Now when } s=0,\ T=W\sin\varphi.$$

$$\therefore T=W\sin\varphi(1-\frac{s}{L}).\quad \text{Also } dl=(1+\frac{T}{M})ds=[1+W\sin\varphi(\frac{1}{M}-\frac{s}{LM})]ds.$$

$$l=\int_0^L[1+W\sin\varphi(\frac{1}{M}-\frac{s}{LM})]ds=L[1+W\sin\varphi(\frac{1}{M}-\frac{1}{2M})]$$